زتونة المجالات

Vector Calculus

Convert Between [POINTS]

- Cylindrical to Cartesian

$$x = \rho \cos\phi \quad ; \quad y = \rho \sin\phi \quad ; \quad z = z$$

- Cartesian to Cylindrical

$$\rho = \sqrt{x^2 + y^2} \quad ; \quad \phi = \tan^{-1}\left(\frac{y}{x}\right) \quad ; \quad z = z$$

- Spherical to Cartesian

$$x = r\sin\theta\cos\phi \quad ; y = r\sin\theta\sin\phi \quad ; z = r\cos\theta$$

- Cartesian to Spherical

$$r = \sqrt{x^2 + y^2 + z^2} \quad ; \quad \theta = \cos^{-1}\left(\frac{z}{r}\right) \quad ; \quad \phi = \tan^{-1}\left(\frac{y}{x}\right)$$

Convert Between [Vectors]

Cartesian-Cylindrical

	$\mathbf{a}_\rho$	$\mathbf{a}_\phi$	$\mathbf{a}_z$
$\mathbf{a}_x \cdot$	$\cos\phi$	$-\sin\phi$	0
$\mathbf{a}_y \cdot$	$\sin\phi$	$\cos\phi$	0
$\mathbf{a}_z \cdot$	0	0	1

Cartesian - Spherical

	$\mathbf{a}_r$	$\mathbf{a}_\theta$	$\mathbf{a}_\phi$
$\mathbf{a}_x \cdot$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\mathbf{a}_y \cdot$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\mathbf{a}_z \cdot$	$\cos\theta$	$-\sin\theta$	0

Scalar triple product

$$\overline{\mathbf{A}}.(\overline{\mathbf{B}} \times \overline{\mathbf{C}}) = \begin{bmatrix} x_A & y_A & z_A \\ x_B & y_B & z_B \\ x_C & y_C & z_C \end{bmatrix}$$

تمثل مساحه متوازي المستطيلات لو بتساوي صفر اذن هم على مستوى واحد

$$\overline{\mathbf{A}}.(\overline{\mathbf{B}} \times \overline{\mathbf{C}}) = \overline{\mathbf{B}}.(\overline{\mathbf{C}} \times \overline{\mathbf{A}}) = \overline{\mathbf{C}}.(\overline{\mathbf{A}} \times \overline{\mathbf{B}})$$

Vector triple product

$$\overline{\mathbf{A}} \times (\overline{\mathbf{B}} \times \overline{\mathbf{C}})$$

Lagrange's formula

$$\overline{\mathbf{A}} \times (\overline{\mathbf{B}} \times \overline{\mathbf{C}}) = \overline{\mathbf{B}}(\overline{\mathbf{A}}.\overline{\mathbf{C}}) - \overline{\mathbf{C}}(\overline{\mathbf{A}}.\overline{\mathbf{B}})$$

Jacobi identity

$$\overline{\mathbf{A}} \times (\overline{\mathbf{B}} \times \overline{\mathbf{C}}) + \overline{\mathbf{B}} \times (\overline{\mathbf{C}} \times \overline{\mathbf{A}}) + \overline{\mathbf{C}} \times (\overline{\mathbf{A}} \times \overline{\mathbf{B}}) = 0$$

$$(\overline{\mathbf{A}} \times \overline{\mathbf{B}}) \times \overline{\mathbf{C}} = \overline{\mathbf{A}} \times (\overline{\mathbf{B}} \times \overline{\mathbf{C}}) - \overline{\mathbf{B}} \times (\overline{\mathbf{A}} \times \overline{\mathbf{C}})$$

Gradient theorem	$\int_a^b \overline{\nabla} T . d\overline{\mathbf{l}} = T(b) - T(a)$
Divergence theorem (Green , Gauss)	$\int_{volume} (\overline{\nabla} . \overline{\mathbf{V}}) dv = \oint_{surface} \overline{\mathbf{V}} . d\overline{s}$
Curl Theorem (Stoke)	$\int_{surface} (\overline{\nabla} \times \overline{\mathbf{V}}) . d\overline{\mathbf{S}} = \oint_{line} \overline{\mathbf{V}} . d\overline{\mathbf{l}}$

Differential Element		
Cartesian	**Cylindrical**	**Spherical**
$dl = dx\overline{\mathbf{a}}_{\mathbf{x}} + dy\overline{\mathbf{a}}_{\mathbf{y}} + dz\overline{\mathbf{a}}_{\mathbf{z}}$	$dl = d\rho\overline{\mathbf{a}}_{\rho} + \rho d\phi\overline{\mathbf{a}}_{\phi} + dz\overline{\mathbf{a}}_{\mathbf{z}}$	$dl = dr\overline{\mathbf{a}}_{\mathbf{r}} + rd\theta\overline{\mathbf{a}}_{\theta} + rsin\theta d\phi\overline{\mathbf{a}}_{\phi}$
$\overline{\mathbf{dA_x}} = dydz\overline{\mathbf{a_x}}$ $\overline{\mathbf{dA_y}} = dzdx\overline{\mathbf{a_y}}$ $\overline{\mathbf{dA_z}} = dxdy\overline{\mathbf{a_z}}$	$\overline{\mathbf{dS_x}} = \rho d\phi dz\overline{\mathbf{a}_{\rho}}$ $\overline{\mathbf{dS_y}} = dzd\rho\overline{\mathbf{a}_{\phi}}$ $\overline{\mathbf{ds_z}} = \rho d\rho d\phi\overline{\mathbf{a_z}}$	$\overline{\mathbf{dS_r}} = r^2 sin\theta . d\theta d\phi\overline{\mathbf{a_r}}$ $\overline{\mathbf{dS}_{\theta}} = r \sin\theta . drd\theta\overline{\mathbf{a}_{\theta}}$ $\overline{\mathbf{dS}_{\phi}} = rdrd\theta\overline{\mathbf{a}_{\phi}}$
$dV = dxdydz$	$dV = \rho d\rho d\phi dz$	t $dV = r^2 sin\theta drd\theta d\phi$

Gradience (scalar) :

$$\overline{\nabla} f = \left(\frac{\partial f}{\partial x}\overline{\mathbf{a_x}} + \frac{\partial f}{\partial y}\overline{\mathbf{a_y}} + \frac{\partial f}{\partial z}\overline{\mathbf{a_z}} \right)$$

Div Laws (Vector)

$$\text{div } \mathbf{D} = \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} \right) \qquad \text{(rectangular)}$$

Curl

$$\overline{\nabla} \times \overline{\mathbf{A}} = \begin{bmatrix} \overline{\mathbf{a}}_x & \overline{\mathbf{a}}_y & \overline{\mathbf{a}}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x_A & y_A & z_A \end{bmatrix}$$

GRADIENT

RECTANGULAR $\nabla V = \frac{\partial V}{\partial x}\mathbf{a}_x + \frac{\partial V}{\partial y}\mathbf{a}_y + \frac{\partial V}{\partial z}\mathbf{a}_z$

CYLINDRICAL $\nabla V = \frac{\partial V}{\partial \rho}\mathbf{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi + \frac{\partial V}{\partial z}\mathbf{a}_z$

SPHERICAL $\nabla V = \frac{\partial V}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\mathbf{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi$

DIVERGENCE

RECTANGULAR $\nabla \cdot \mathbf{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$

CYLINDRICAL $\nabla \cdot \mathbf{D} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}$

SPHERICAL $\nabla \cdot \mathbf{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(D_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial \phi}$

CURL

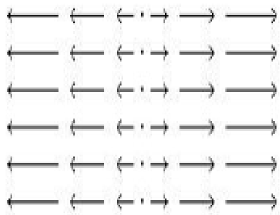
RECTANGULAR $\nabla \times \mathbf{H} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\mathbf{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\mathbf{a}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\mathbf{a}_z$

CYLINDRICAL $\nabla \times \mathbf{H} = \left(\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z}\right)\mathbf{a}_\rho + \left(\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho}\right)\mathbf{a}_\phi$

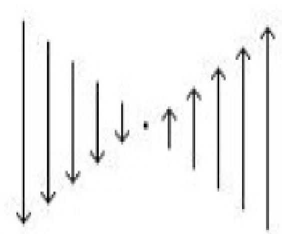
$$+ \frac{1}{\rho}\left[\frac{\partial(\rho H_\phi)}{\partial \rho} - \frac{\partial H_\rho}{\partial \phi}\right]\mathbf{a}_z$$

SPHERICAL $\nabla \times \mathbf{H} = \frac{1}{r\sin\theta}\left[\frac{\partial(H_\phi \sin\theta)}{\partial \theta} - \frac{\partial H_\theta}{\partial \phi}\right]\mathbf{a}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial H_r}{\partial \phi} - \frac{\partial(rH_\phi)}{\partial r}\right]\mathbf{a}_\theta$

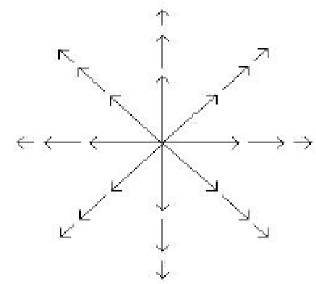
$$+ \frac{1}{r}\left[\frac{\partial(rH_\theta)}{\partial r} - \frac{\partial H_r}{\partial \theta}\right]\mathbf{a}_\phi$$



Case 1: A vector field with positive divergence but zero curl.



Case 2: A vector field with nonzero curl (the curl vector points out of the computer screen), but zero divergence.



Case 3: An inverse square field. Its divergence has a value and curl is zero.

	Case 1	**Case 2**	**Case 3**
$\boldsymbol{Divergence}$	$\overline{\boldsymbol{\nabla}}.\overline{\boldsymbol{E}} \neq \mathbf{0}$	$\overline{\boldsymbol{\nabla}}.\overline{\boldsymbol{E}} = \mathbf{0}$	$\overline{\boldsymbol{\nabla}}.\overline{\boldsymbol{E}} \neq \mathbf{0}$
$\boldsymbol{Curl}$	$\overline{\boldsymbol{\nabla}} \times \overline{\boldsymbol{E}} = \mathbf{0}$	$\overline{\boldsymbol{\nabla}} \times \overline{\boldsymbol{E}} \neq \mathbf{0}$	$\overline{\boldsymbol{\nabla}} \times \overline{\boldsymbol{E}} = \mathbf{0}$

خلاصة القول اللي تفهم بيه طريقه ال Div وال Curl تخيل الأسهم هي حركة موجات ميه والنقطه اللي بتدرس عندها فيها كورة بينج ، لو الكورة هتتحرك أي حته فوق تحت يمين شمال يبقى في Div موجود ، لو هتلف حوالين نفسها يبقى في curl لو هتتحرك وتلف حوالين نفسها الاتنين يبقى في الاتنين لو الموجات بتلاشي بعض وهتفضل ثابته ومش هتتحرك يبقى مفيش حاجه

Symbol	Name	Unit	Abbreviation
v	Velocity	meter/second	m/s
F	Force	newton	N
Q	Charge	coulomb	C
r, R	Distance	meter	m
ϵ_0, ϵ	Permittivity	farad/meter	F/m
E	Electric field intensity	volt/meter	V/m
ρ_v	Volume charge density	coulomb/meter^3	C/m^3
v	Volume	meter^3	m^3
ρ_L	Linear charge density	coulomb/meter	C/m
ρ_S	Surface charge density	coulomb/meter^2	C/m^2
Ψ	Electric flux	coulomb	C
D	Electric flux density	coulomb/meter^2	C/m^2
S	Area	meter^2	m^2
W	Work, energy	joule	J
L	Length	meter	m
V	Potential	volt	V
p	Dipole moment	coulomb-meter	C·m
I	Current	ampere	A
J	Current density	ampere/meter^2	A/m^2
μ_e, μ_h	Mobility	meter^2/volt-second	m^2/V·s
e	Electronic charge	coulomb	C
σ	Conductivity	siemens/meter	S/m
R	Resistance	ohm	Ω
P	Polarization	coulomb/meter^2	C/m^2
$\chi_{e,m}$	Susceptibility		
C	Capacitance	farad	F
R_s	Sheet resistance	ohm per square	Ω
H	Magnetic field intensity	ampere/meter	A/m
K	Surface current density	ampere/meter	A/m
B	Magnetic flux density	tesla (or weber/meter^2)	T (or Wb/m^2)
μ_0, μ	Permeability	henry/meter	H/m
Φ	Magnetic flux	weber	Wb
V_m	Magnetic scalar potential	ampere	A
A	Vector magnetic potential	weber/meter	Wb/m
T	Torque	newton-meter	N·m
m	Magnetic moment	ampere-meter^2	A·m^2
M	Magnetization	ampere/meter	A/m
$\mathcal{R}$	Reluctance	ampere-turn/weber	A·t/Wb
L	Inductance	henry	H
M	Mutual inductance	henry	H

Symbol	Name	Unit	Abbreviation
ω	Radian frequency	radian/second	rad/s
c	Velocity of light	meter/second	m/s
λ	Wavelength	meter	m
η	Intrinsic impedance	ohm	Ω
k	Wave number	meter^{-1}	m^{-1}
α	Attenuation constant	neper/meter	Np/m
β	Phase constant	radian/meter	rad/m
f	Frequency	hertz	Hz
S	Poynting vector	watt/meter^2	W/m^2
P	Power	watt	W
δ	Skin depth	meter	m
Γ	Reflection coefficient		
s	Standing wave ratio		
γ	Propagation constant	meter^{-1}	m^{-1}
G	Conductance	siemen	S
Z	Impedance	ohm	Ω
Y	Admittance	siemen	S
Q	Quality factor		

ELECTRIC FIELDS

Columbs Law :

$$\bar{F} = \frac{(Q_1 Q_2)}{4\pi\epsilon_0 R^2}\ \overline{a_R} == \frac{(Q_1 Q_2)}{4\pi\epsilon_0 R^3}\ \bar{R}$$

Electric Field Laws

Point Charge	Line charge	Infinit line	
$E = \frac{Q}{4\pi\epsilon_0 R^3}\ \bar{R}$	$E = \int \frac{\rho_L\, dL}{4\pi\epsilon_0 R^2}\ \overline{a_R}$	$\bar{E} = \frac{\rho_L}{2\pi\epsilon_0 R}\ \overline{a_R}$	
Ring	Disk	Surface	Infinite surface
$E = \frac{\rho_0 a h}{2\epsilon_0 (a^2 + h^2)^{\frac{3}{2}}}\ \hat{a}_z$	$E = \frac{\rho_s}{2\epsilon_0}\left[1 - \frac{h}{\sqrt{h^2 + R^2}}\right]$	$E = \int \frac{\rho_S ds}{4\pi\epsilon_0 R^2}\ \overline{a_R}$	$E = \frac{\rho_s}{2\epsilon_0}$

Electric Flux – Flux density – gauss law

$$\psi = Q_{enc} = \oiint \bar{D}\ \ d\,\bar{s}$$

علاقه D بال E

$$D = \epsilon E \Rightarrow \frac{D}{E} = \epsilon \quad \therefore \quad D = \frac{E}{\epsilon}$$

Electric Density (D) relations

Point Charge	Infinit line	Infinite surface	Cylindrical volume
$D = \frac{Q}{4\pi R^3}\ \bar{R}$	$\bar{D} = \frac{\rho_L}{2\pi R}\ \overline{a_R}$	$D = \frac{\rho_s}{2}$	$\rho < a$ $D = \frac{\rho_v}{2}\ \rho$ $\rho > a$ $\frac{\rho_v\left(\frac{a^2}{2}\right)}{\rho}$
Finite cylindrical surface	Finite spherical shell surface	Surface	
$D = \frac{\rho_s}{\rho}\ a$	$D = \rho_s \frac{a^2}{r^2}$	$D = \int \frac{\rho_S ds}{4\pi R^2}\ \overline{a_R}$	

Dot form of charge density

From green's or gauss theorem

$$\int_{volume} (\nabla . \mathbf{V}) dv = \oint_{surface} \mathbf{V} . d\mathbf{s}$$

$$\therefore \quad \oiint \bar{D}.\,d\bar{s} = \int \overline{\boldsymbol{\nabla}} \cdot \overline{\boldsymbol{D}}\; d\bar{v} = \int \boldsymbol{\rho_v}\; d\bar{v}$$

$\therefore \quad \overline{\boldsymbol{\nabla}} \cdot \overline{\boldsymbol{D}} = \boldsymbol{\rho_v}$ **Maxwell's 1st equation**

Energy and potential

$$W = -\int_{initial}^{final} q\,\bar{E}.d\bar{L}$$

$$Note\ :\ W_{AB} = -W_{BA} \quad ; \quad W_{ABCDA} = 0$$

$$\boldsymbol{V_{AB} = -\int_{B}^{A} \bar{E}\,.d\bar{L}}$$

$$\oint \bar{E}\cdot d\bar{L} = 0$$
$$\bar{\nabla}\times\bar{\boldsymbol{E}} = \mathbf{0}$$

Maxwell's 2nd equation (assuming conservative system)

$$\oint \bar{E}\cdot d\bar{L} = \bar{\nabla}\times\bar{\boldsymbol{E}} \neq \mathbf{0}\ \ (\boldsymbol{non-conservative\ system})$$

Potential difference Equations :

Point Charge	Absolute potential	Line charge /Finite	Infinite line	Ring
$V_{AB} = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r_A}-\frac{1}{r_B}\right]$	$V_A = V_{A\infty} = \frac{Q}{4\pi\epsilon_0 r_A}$	$V = \int \frac{\rho_L dL}{4\pi\epsilon_0 r} = \frac{\rho_L}{2\pi\epsilon_0}\ln\frac{L+\sqrt{L^2+a^2}}{a}$	$V_{AB} = \frac{\rho_L}{2\pi\epsilon_0}\ln\frac{\rho_B}{\rho_A}$	$V = \frac{\rho_L a}{2\epsilon_0\sqrt{a^2+h^2}}$

Equipotential surfaces : surfaces that has the same potential

$$\frac{\Delta V}{\Delta L} = Potential\ gradient$$
$$V = -\int \bar{E}.d\bar{L}$$
$$\Delta V = -E\,\Delta L$$

كمعيار

$$\therefore\ E = -\frac{\Delta V}{\Delta L}$$

كمتجه

$$\bar{\boldsymbol{E}} = -\boldsymbol{\nabla V}$$

To move a charge from infinity to a point in the free space with other charges affect that point with a field

$$W_E = \frac{1}{2}\sum_{m=1}^{n} Q_m V_m$$
$$W_E = \frac{1}{2}\int \bar{D}.\bar{E}\ \ dV \qquad (J)$$

$$W_E = \frac{1}{2}\bar{D}.\bar{E} \qquad \left(\frac{J}{m^3}\right)$$

Electrostatic Fields in materials

يمكننا تقسيم الشحنات لشحنه سطحيه و حجميه ، في الموصل نتعامل على ان السلك اسطوانه يمر بها شحنه حجميه

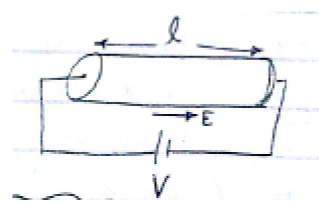
العوامل اللي بتكون التيار (كثافه الشحنات ، مساحه المقطع ، طوله ، الزمن (سرعه الموصلات))

$$i = \frac{\rho_v \Delta S \Delta L}{\Delta t} \quad (\,divide\ by\ area\,)$$
$$\boldsymbol{J = \rho_v\ v_d}$$

لا تعتمد على نوع توصيليه الماده ، تعتمد على نوع التوصيل conduction , convention

Ohm's Law

$$J = \sigma \bar{E}$$



$$V = -\int E.dL = E.L$$
$$ohm's\ law\ J = \sigma E \Rightarrow \frac{I}{S} = \sigma\frac{V}{L} \Rightarrow \frac{V}{I} = \frac{l}{\sigma A} = R$$
$$for\ non\,uniform\,V:\ R = \frac{V}{I} = \frac{-\int E.dL}{\int_s \sigma\, E.dS}$$

Field Effect on Dielectric Materials

يتم استقطاب الماده عند وجود مجال كهربي عليها

$$P = \chi_e\, \epsilon_0\, \bar{E}$$
$$\chi_e = \epsilon_r - 1$$

Dielectric Strenght : اقل مجال كهربي يكسر العزم

$$D = \epsilon E = \epsilon_0 \epsilon_r E = \epsilon_0\,(1+\chi_e)E = P + \epsilon_0 E$$

خواص المواد العازلة : (لا يوجد عزل مثالي

المادة العازلة لا تحتوي الكترونات حرة

عند التاثير على المواد العازلة يتم استقطابها

Boundary Conditions

Between 1- Dielectric – Dielectric 2- Dielectric –Conductor

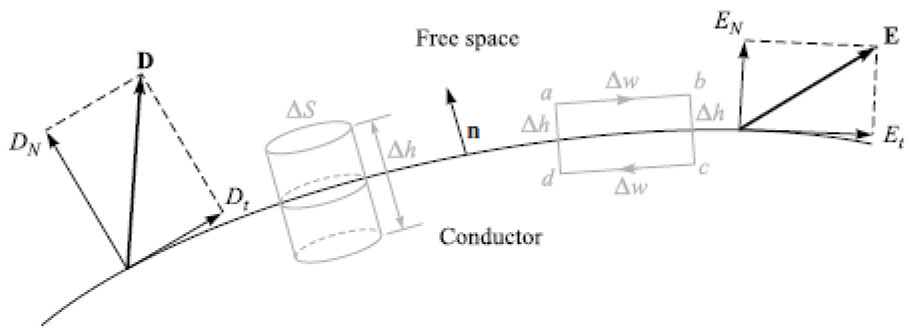


Figure 5.4 An appropriate closed path and gaussian surface are used to determine boundary conditions at a boundary between a conductor and free space; $E_t = 0$ and $D_N = \rho_S$.

1- Dielectric – Dielectric

Normal	Tangent
Gauss (ρ_s الشحنه على السطح الفاصل بين المادتين $\oiint D.dS = Q_{enc}$ $D_{1n}\Delta S - D_{2n}\,\Delta S = \rho_s\,\Delta S$ $D_{1n} - D_{2n} = \rho_s$ $if\ \rho_s = 0$ $\therefore\ \boldsymbol{D_{1n} = D_{2n}}$	Ampere's law $\oint E.dL = 0$ $\therefore E_{1t}\Delta w - E_{1n}\frac{\Delta h}{2} - E_{2n}\frac{\Delta h}{2} - E_{2t}\Delta w + E_{2n}\frac{\Delta h}{2} + E_{2n}\frac{\Delta h}{2} = 0$ $\therefore\ \boldsymbol{E_{1t} = E_{2t}}$

2- Conductor – Dielectric

Normal	Tangent
Gauss $\oiint D.dS = Q_{enc}$ $\therefore \boldsymbol{D_n = \rho_s}$	$\boldsymbol{E_t = 0}$

Capacitance

$$C = \frac{Q}{V} \quad \left(\left(if\ E\ \ (electric\ field\ is\ uniform)\right)\right)$$

$$W_E = \frac{1}{2} C\, V^2 \quad ; \boldsymbol{Energy\ Density = W = \frac{W_E}{AD} = \frac{1}{2}\epsilon_0 E^2}$$

ملحوظات للمكثفات

1- هيجي مساله مكثفات وش

2- المكثفات على التوالي

$$C_t = \frac{C_1 C_2}{C_1 + C_2}$$

المكثفات على التوازي

$$C_t = C_1 + C_2$$

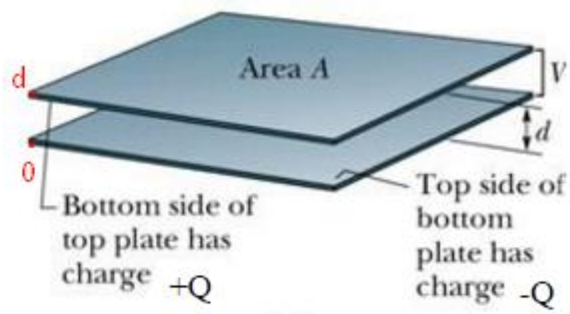
3- أنواع المكثفات ممكن يكون

Parallel plate– Coaxial (cylindrical) – Spherical

وركز في شكل المكثف والعدد في حاله اسطواني او كروي مش مستطيل

استنتاج قانون المكثف :

Parallel Plate :



(a)

$$C = \frac{Q}{V}$$

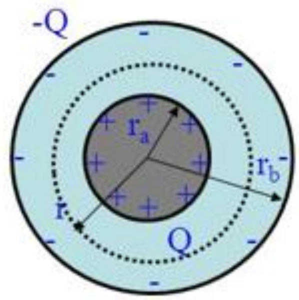
$$V = -\int_d^0 E.dL$$

$$V = -\int_d^0 \frac{\rho_s}{\epsilon}.dL$$

$$\therefore V = \frac{\rho_s}{\epsilon} d$$

$$\therefore C = \frac{Q}{V} = \frac{\rho_s A}{\frac{\rho_s}{\epsilon} d} = \frac{\epsilon A}{d}$$

Spherical



$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{Q}{\varepsilon_0} \rightarrow E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

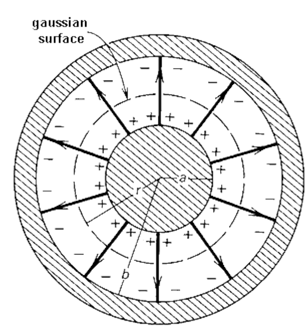
$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

$$V_{ab} = V_a - V_b = \frac{Q}{4\pi\varepsilon_0 r_a} - \frac{Q}{4\pi\varepsilon_0 r_b} = \frac{Q}{4\pi\varepsilon_0} \frac{r_b - r_a}{r_a r_b}$$

$$\boxed{C = \frac{Q}{V_{ab}} = 4\pi\varepsilon_0 \frac{r_a r_b}{r_b - r_a}}$$

Cylindrical (Co-axial)

$$\oint E.ds = \frac{Q_{enc}}{\epsilon_0} \Rightarrow E.2\pi rL = \frac{Q}{\epsilon_0}$$



$$E = \frac{Q}{2\pi\epsilon_0 rL}$$

$$V = \int_a^b E.dr = \int_a^b \frac{Q}{2\pi\epsilon_0 rL} dr = \frac{Q}{2\pi\epsilon_0 L}\int_a^b r\,dr$$

$$V = \frac{Q}{2\pi L}\ln\left(\frac{b}{a}\right)$$

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{2\pi L}\ln\left(\frac{b}{a}\right)} = \frac{\mathbf{2\pi\epsilon_0 L}}{\mathbf{\ln\left(\frac{b}{a}\right)}}$$

Magnetic Fields

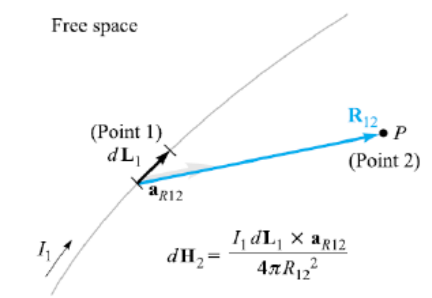
The Sources of Magnetic Field :

1-Permanent magnet 2-Linearly changing electric field w/ time 3- a direct current

Biot-Savart

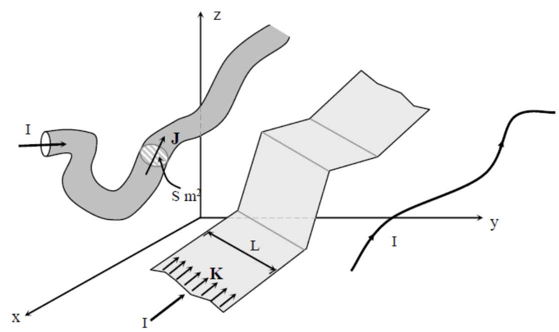
$$\boldsymbol{H} \propto \frac{\boldsymbol{m_1 m_2}}{\boldsymbol{R^2}}$$ القوة المؤثرة على وحده الاقطاب المغناطيسية

$$d\overline{\boldsymbol{H}} = \frac{\boldsymbol{Id\overline{L} \times \overline{a_R}}}{\boldsymbol{4\pi R^2}} = \frac{\boldsymbol{Id\overline{L} \times \overline{R}}}{\boldsymbol{4\pi R^3}}$$



The total current crossing any closed surface is zero. It is this current flowing in a closed circuit that must be our experimental source(In a closed path), not the differential element. It follows that only the integral form of the Biot-Savart law can be verified experimentally, $\overline{H} = \oint \left(\frac{\boldsymbol{Id\overline{L} \times \overline{a_R}}}{\boldsymbol{4\pi R^2}}\right)$

التيار الكهربي هو اهم مصادر المجال المغناطيسي عشان كده لازم ندرس اشكاله المختلفه



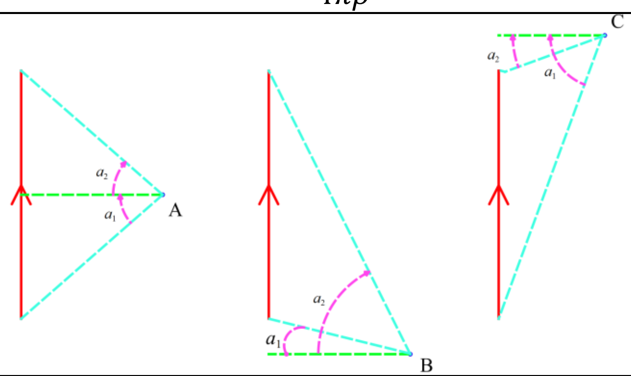
$Surface\ current\ density : I = K * L\ (uniform)\ or\ I = \int Kdl$

the same with

$Id\overline{L} = Kd\overline{s} = Jd\overline{V}$

$$H = \int_s \frac{KdS \times a_R}{4\pi R^2} = \int_v \frac{Jdv \times a_R}{4\pi R^2}$$

H from :

Infinite Line	Line
$\overline{H} = \frac{I}{2\pi\rho}\ \overline{a_\phi}$	$\overline{H} = \frac{I}{4\pi\rho}[\sin(\alpha_2) - \sin(\alpha_1)]\overline{a_\phi}$
	

H from Coaxial cable

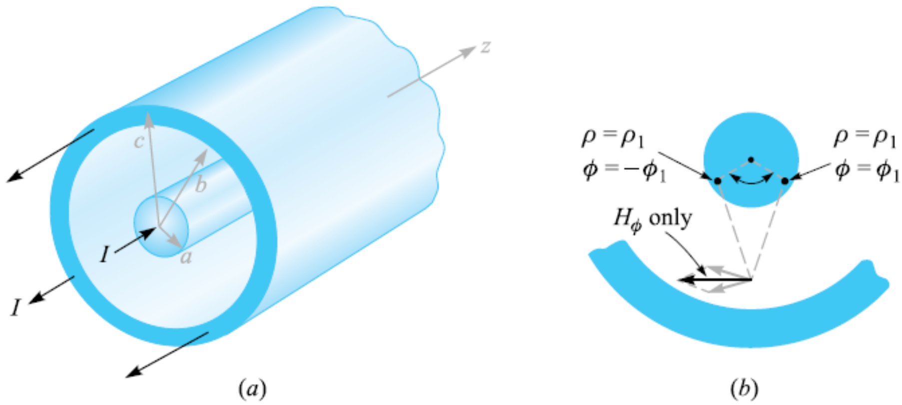


Figure 7.8 (*a*) Cross section of a coaxial cable carrying a uniformly distributed current I in the inner conductor and $-I$ in the outer conductor. The magnetic field at any point is most easily determined by applying Ampère's circuital law about a circular path. (*b*) Current filaments at $\rho = \rho_1, \phi = \pm\phi_1$, produces $\mathbf{H}_\rho$ components which cancel. For the total field, $\mathbf{H} = H_\phi \mathbf{a}_\phi$.

بتطبيق قانون امبير (شبه قانون جاوس في الكهربية)

$\rho < a$	$a < \rho < b$	$b < \rho < c$	$\rho > c$
$I_{\text{encl}} = I\dfrac{\rho^2}{a^2}$ $2\pi\rho H_\phi = I\dfrac{\rho^2}{a^2}$ $H_\phi = \dfrac{I\rho}{2\pi a^2} \quad (\rho < a)$	$H_\phi 2\pi\rho = I$ $H_\phi = \dfrac{I}{2\pi\rho}$	$2\pi\rho H_\phi = I - I\left(\dfrac{\rho^2 - b^2}{c^2 - b^2}\right)$ $H_\phi = \dfrac{I}{2\pi\rho}\dfrac{c^2-\rho^2}{c^2-b^2} \quad (b < \rho < c)$	$H_\phi = 0 \quad (\rho > c)$

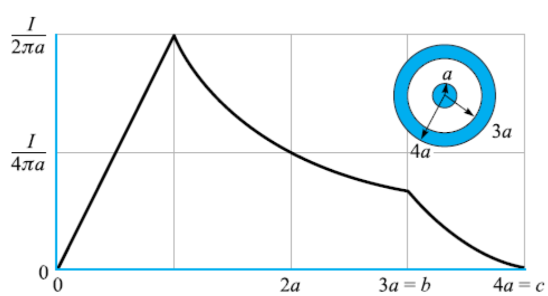


Figure 7.9 The magnetic field intensity as a function of radius in an infinitely long coaxial transmission line with the dimensions shown.

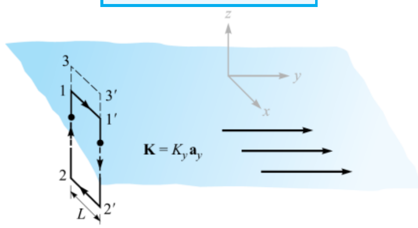
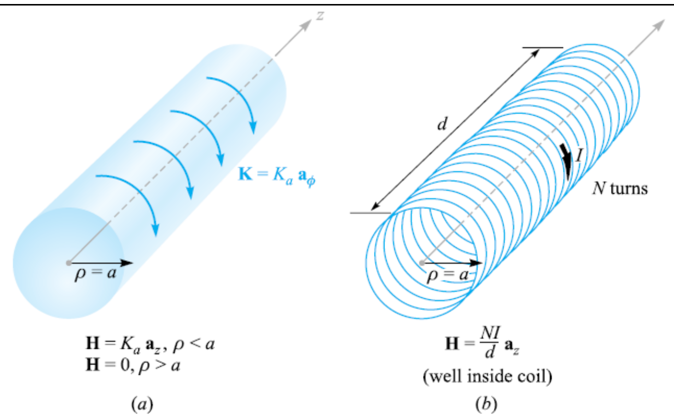
<table>
<tr><td></td><td></td></tr>
<tr><td>لاي نقطه على احد الجانبين
$$\boxed{\mathbf{H} = \frac{1}{2}\mathbf{K} \times \mathbf{a}_N}$$

</td><td>H from a surface</td></tr>
<tr><td>

Figure 7.11 (a) An ideal solenoid of infinite length with a circular current sheet $\mathbf{K} = K_a \mathbf{a}_\phi$. (b) An N-turn solenoid of finite length d.
لو محدود
$$H = \frac{nI}{2}\left[\cos\theta_2 - \cos\theta_1\right]$$</td><td>H from a solonoid</td></tr>
</table>

H from a troid

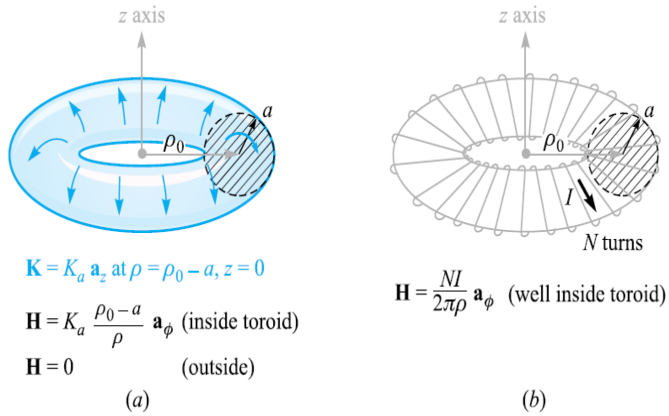


Figure 7.12 (*a*) An ideal toroid carrying a surface current K in the direction shown. (*b*) An *N*-turn toroid carrying a filamentary current *I*.

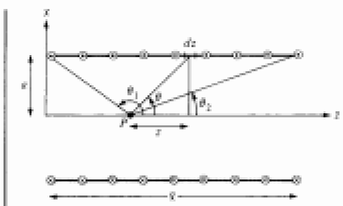


Figure 7.9 For Example 7.4; cross section of a solenoid.

Solution:

Consider the cross section of the solenoid as shown in Figure 7.9. Since the solenoid consists of circular loops, we apply the result of Example 7.3. The contribution to the magnetic field H at P by an element of the solenoid of length dz is

$$dH_z = \frac{I\,dl\,a^2}{2[a^2 + z^2]^{3/2}} = \frac{Ia^2 n\,dz}{2[a^2 + z^2]^{3/2}}$$

where $dl = n\,dz = (N/\ell)\,dz$. From Figure 7.9, $\tan\theta = a/z$; that is,

$$dz = -a\,\text{cosec}^2\,\theta\,d\theta = -\frac{[z^2 + a^2]^{3/2}}{a^2}\sin\theta\,d\theta$$

Hence,

$$dH_z = -\frac{nI}{2}\sin\theta\,d\theta$$

or

$$H_z = -\frac{nI}{2}\int_{\theta_1}^{\theta_2}\sin\theta\,d\theta$$

Thus

$$\mathbf{H} = \frac{nI}{2}(\cos\theta_2 - \cos\theta_1)\,\mathbf{a}_z$$

as required. Substituting $n = N/\ell$ gives

$$\mathbf{H} = \frac{NI}{2\ell}(\cos\theta_2 - \cos\theta_1)\,\mathbf{a}_z$$

At the center of the solenoid,

$$\cos\theta_2 = \frac{\ell/2}{[a^2 + \ell^2/4]^{1/2}} = -\cos\theta_1$$

and

$$\mathbf{H} = \frac{In\ell}{2[a^2 + \ell^2/4]^{1/2}}\,\mathbf{a}_z$$

If $\ell \gg a$ or $\theta_2 \simeq 0°$, $\theta_1 \simeq 180°$,

$$\mathbf{H} = nI\mathbf{a}_z = \frac{NI}{\ell}\,\mathbf{a}_z$$

Force from a Magnetic Field and Electric Field

As shown from the direction of H the Force from a magnetic field is in Right عموديه angle with the velocity of the particle

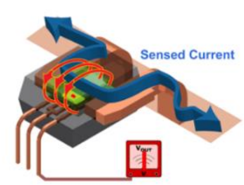
Which means : Magnetic Force can never change the velocity of a moving particle ; this means that <u>magnetic field</u> is incapable of transferring Energy to the moving charge

$$F_e(Electric\ Force) = Q\bar{E}$$
$$F_m(Magnetic\ Force) = Q\ \bar{V} \times \bar{B}$$
$$F_{total}(Superposition\ force) = Q(\bar{E} + \bar{V} \times \bar{B})$$

Notes : a large number of very small forces $dF = dQ\ V \times B$ can cause electrons to shift slightly and then the conductor as a whole , it causes displacement in center of gravity between +ve and –ve charges so the columb (electric) force tend to resist this displacement>

What's Current sensor (Hall Voltage)

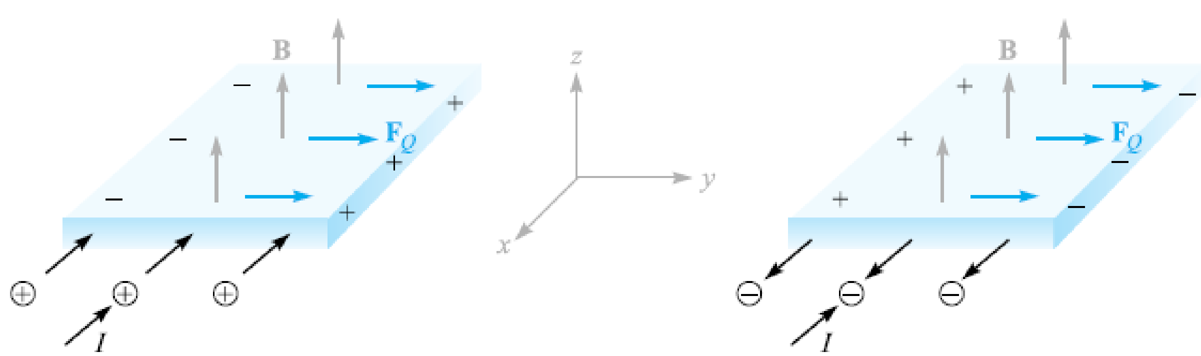
It's a method to differentiate between the electrons and holes current by comparing their hall voltage in whats called current sensor this helps to determine if the semiconductor is n-type or p-type



:

.

The charge separation that does result, however, is disclosed by the presence of a slight potential difference across the conductor sample in a direction perpendicular to both the magnetic field and the velocity of the charges.
The voltage is known as the Hall voltage, and the effect itself is called the Hall effect.



Force on a current-carrying conductor

$$J = \rho_V \bar{v} \;\; and \quad Q = \rho_V dV$$

$$dF = dQ\ \bar{v} \times \bar{B} = \rho_V\ dV \frac{J}{\rho_V} \times \bar{B} = Jd\bar{V} \times \bar{B} \ = (\bar{J} \times \bar{B}\ )dV$$

$$the\ same \quad dF = \ (\bar{J} \times \bar{B}\ )dV = \ (\bar{K} \times \bar{B}\ )dS = \ (\bar{I} \times \bar{B}\ )dL = Id\bar{L} \times \bar{B}$$

$$\bar{F} \ = \int_v (\bar{J} \times \bar{B}\ )dV = \int_S \ (\bar{K} \times \bar{B}\ )dS$$

$$\bar{F} = \oint Id\bar{L} \times \bar{B} = \ -I \oint \bar{B} \times d\bar{L} = I\ \bar{L} \times \bar{B}$$

$$|F| = BIL\sin(\theta)$$

We can express Force between 2 current elements without the magnetic field but it will be complex as follows:

$$from:\ dF = I_2 d\overline{L_2} \times \bar{B} \ \ \& \ \ d\bar{H} = \frac{I_1 dL_1 \times a_R}{4\pi R^2} \ \ \& \ B = \mu_0 H$$

$$\therefore F_2 = \mu_0 \frac{I_1 I_2}{4\pi} \quad \oint [\ dL_2 \times \ \oint \frac{dL_1 \times a_R}{R^2}\ ] \rightarrow \ \ (\ very\ very\ complex\ )$$

<u>**NOTE :** the total force on a closed current loop in a uniform = 0</u>

TORQUE

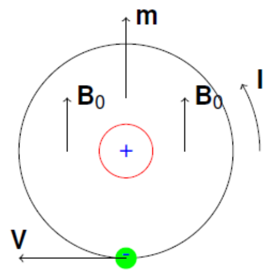
$$\bar{T} = \bar{R} \times \bar{F}$$

For a closed loop

$$dT = IdS \times B$$
$$define \;\; dm = IdS$$
$$\therefore \;\; dT = dm \times B \quad (dipole\ moment\ A.m^2)$$
$$T = m \times B = IS \times B$$

<u>The torque direction is that to align the magnetic field produced by the loop with the applied magnetic field causing the torque</u>

في محاوله هنعتبر الالكترون حوالين النواه يمثل لوب تياره عكس اتجاه حركه الالكترون وتم تسليط فيض مغناطيسي خارجي ده هيسبب ان التيار بتاع حركه الالكترونات الصغير ده هيولد مجال مغناطيسي يحاول يحرك النواه في اتجاه المجال الخارجي لحد ما يبقوا aligned ويساعدو بعض - او يضادوا ! – على حسب



لهذا السبب

1- المجال المغناطيسي داخل المادة هيكون اكبر منه خارج الماده بسبب ان الالكترونات هتولد مجال إضافي داخلها
2- ال spin magnetic momentبتاع الالكترون بتساوي $\pm 9 \times 10^{-24} Am^2$ موجب لو بتساعد سالب لو بتضاد المجال الخارجي
3- الالكترونات اللي بتساهم بمجال مغناطيسي فقط الالكترونات اللي في المدارات الخارجيه غير المكتمله
4- طبعا كل الكلام اللي فوق ده محتاج معادلات من ميكانيكا الكم و حورات المهم اللي نستنجه ان نوع الماده وعدد الالكترونات و اللي بنكلم فيه ده جزء صغيييير جدا لان الماده مليانه momentsلاسباب مختلفه وكلها بتاثر على تصنيف الماده المغناطيسيه الا ان التبسيط اللي فوق ده هو الأساس العلمي لاجهزه الMRI ربنا لا يدخلكم في واحده

Types Of Material

يمكن تصنيف المواد على حسب اذا كانت المومنت المحصله للالكترونات بتزود الفيض داخلها ام تقلله ام ماذا

Table 8.1 Characteristics of magnetic materials

Classification	Magnetic Moments	B Values	Comments
Diamagnetic	$\mathbf{m}_{orb} + \mathbf{m}_{spin} = 0$	$B_{int} < B_{appl}$	$B_{int} \doteq B_{appl}$
Paramagnetic	$\mathbf{m}_{orb} + \mathbf{m}_{spin} =$ small	$B_{int} > B_{appl}$	$B_{int} \doteq B_{appl}$
Ferromagnetic	$\lvert\mathbf{m}_{spin}\rvert \gg \lvert\mathbf{m}_{orb}\rvert$	$B_{int} \gg B_{appl}$	Domains
Antiferromagnetic	$\lvert\mathbf{m}_{spin}\rvert \gg \lvert\mathbf{m}_{orb}\rvert$	$B_{int} \doteq B_{appl}$	Adjacent moments oppose
Ferrimagnetic	$\lvert\mathbf{m}_{spin}\rvert \gg \lvert\mathbf{m}_{orb}\rvert$	$B_{int} > B_{appl}$	Unequal adjacent moments oppose; low σ
Superparamagnetic	$\lvert\mathbf{m}_{spin}\rvert \gg \lvert\mathbf{m}_{orb}\rvert$	$B_{int} > B_{appl}$	Nonmagnetic matrix; recording tapes

اذن يمكننا تعريف 3 تيارات

1- I وهو التيار الذي نعرفه ناتج عن حركه الكترونات حره في موصل وفي قانون امبير $\oint H.dL = I$

2- التيار الناتج عن ال Bound charge واللذي يعرف I_B وناتج عن المغناطيسية $\oint M.dL = I_B$

3- التيار الكلي وهو مجموع $I_T = I_B + I$ ومن قانون امبير $\oint \left(\frac{B}{\mu_0}\right).dL = I_T$

منها نستنتج عده قوانين هامه

$$\frac{B}{\mu_0} = M + H \qquad \therefore \quad B = \mu_0\,(H + M\,)$$

$$\oint M.dL = I_B = \int_S J_B ds \qquad \therefore \quad \bar{\nabla} \times \bar{M} = \bar{J_B}$$

$$\oint H.dL = I \; = \int_S J ds \qquad \therefore \quad \bar{\nabla} \times \bar{H} = \bar{J}$$

$$\oint \left(\frac{B}{\mu_0}\right).dL = I_T \; = \int_S J_T ds \qquad \therefore \quad \bar{\nabla} \times \frac{\bar{B}}{\mu_0} = \bar{J_T}$$

$$where\ \bar{M} = atoms/m^3 \ * (dipole\ moment\ /atom) = \frac{B}{\mu_0}\ \left(\frac{\chi_m}{1+\chi_m}\right)$$

For Linear isotropic material (if χ_m magnetic susceptibility) $M = \chi_m \; H$

$\therefore \quad B = \mu_0 \, (H + \chi_m H) = \mu_0 \quad (1 + \chi_m) H = \mu_0 \mu_r \; H \qquad where \; \mu_r = (1 + \chi_m)$

$define \; \mu = \mu_r \mu_0$

$then \qquad \boldsymbol{B} = \boldsymbol{\mu H}$

Generally :

As,

$$\overline{\mathbf{B}} = \mu \overline{\mathbf{H}}$$

for homogeneous, linear, isotropic magnetic material that may be described in terms of a relative permeability μ_r

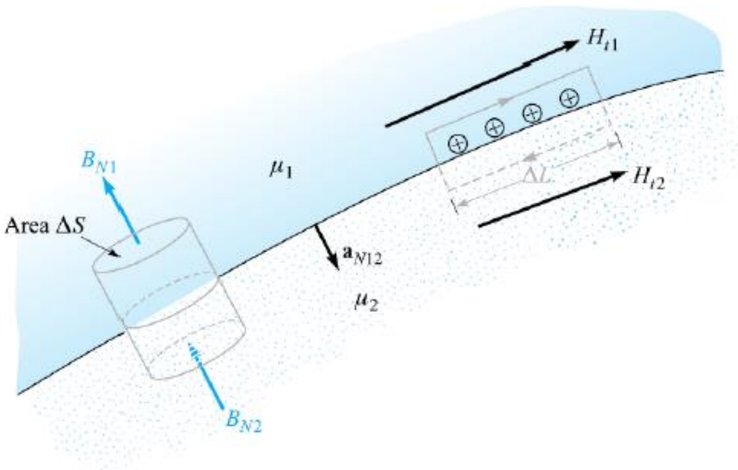
$$\begin{bmatrix} B_x \\ B_y \\ B_Z \end{bmatrix} = \mu \begin{bmatrix} H_x \\ H_y \\ H_Z \end{bmatrix}$$

For anisotropic materials

$$\begin{bmatrix} B_x \\ B_y \\ B_Z \end{bmatrix} = \begin{bmatrix} \mu_{xx} & \mu_{xy} & \mu_{xz} \\ \mu_{yx} & \mu_{yy} & \mu_{yz} \\ \mu_{zx} & \mu_{zy} & \mu_{zz} \end{bmatrix} \begin{bmatrix} H_x \\ H_y \\ H_Z \end{bmatrix}$$

الماتريكس المربعه اسمها tensor

Boundary Conditions

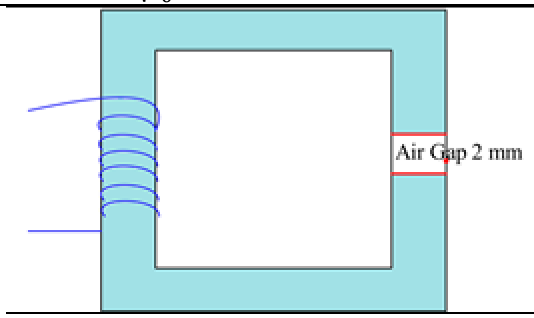
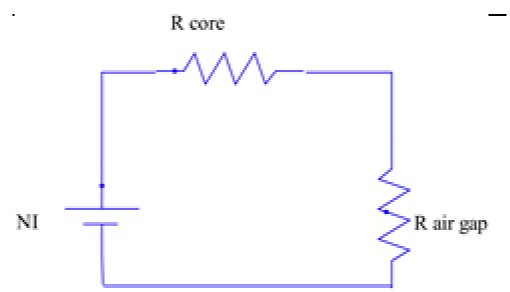


Normal	Tangent
$B_{N1} = B_{N2}$ $\mu_1 H_{N1} = \mu_2 H_{N_2}$ $\frac{\mu_1 M_{N_1}}{\chi_{m1}} = \frac{\mu_2 M_{N_2}}{\chi_{m2}}$	$H_{t1} - H_{t2} = K$ $(H_{t1} - H_{t2}) \times a_{N12} = K$ $(Ht_1 - H_{t2}) = K \times a_{N12}$ $\frac{B_{t1}}{\mu_1} - \frac{B_{t2}}{\mu_2} = K$ $\frac{M_{t1}}{\chi_{m1}} - \frac{M_{t2}}{\chi_{m2}} = K$ Where K (surface current) K = 0 if either materials is conductor

MAGNETIC CIRCUIT

طبعا الجزئية دي احنا فاهمين تطبيقها من التحويل لكن الأساس الفيزيائي العلمي واضح من مقارنه المعادلات بين الكهربية والمغناطيسية واللي بيساعدنا في تقريب الدائرة المغناطيسية لدائرة كهربية ذات تيار مستمر ومقاومات فقط

MAGNETIC	ELECTRIC
1- $H = -\nabla V_m$	1- $E = -\nabla V$
2- $V_{m\,AB} = \int_A^B H.dL$	2- $V_{AB} = \int_A^B E.dL$
3- $B = \mu H$	3- $J = \sigma \bar{E}$
4- $\phi = \int B.ds$	4- $I = \int J.dS$
5- $V_m = \phi \Re$	5- $V = IR$
6- $\Re = \frac{d}{\mu A}$	6- $R = \frac{d}{\sigma A}$
7- $\oint E.dL = 0$	7- $\oint H.dL = NI$
8- $W = \frac{1}{2}\int_v \bar{D}.\bar{E}\ dv$	8- $W = \frac{1}{2}\int_c \bar{B}.\bar{H} dv$ $W = \frac{1}{2}\int_v \mu H^2\, dv = \frac{1}{2}\int_v \frac{B^2}{\mu}\, dv$

Magnetic	**Electrical**
$Mmf = NI = \phi R = Hl$	$Emf = V = IR$
$R = \dfrac{l}{\mu A}$ $\mu = \mu_0 \mu_r$ $\mu_0 = 4\pi \times 10^{-7}$	$R = \dfrac{l}{\sigma A}$
	

Inductance

Flux Linkage ($\boldsymbol{\lambda}$) : the total flux linking the turns of a coil $\lambda = N\phi$

المفروض عند حساب ال flux linkage نحسب فيض كل لفه الأول بس بدل ما نعمل كده بنستخدم ال winding factor and pitch factor لتصحيح الخطا في استخدام المعادله البسيطه

$$\boldsymbol{Inductance\ (L): the\ ration\ between\ the\ flux\ linkage\ and\ the\ current\ it's\ linking}$$

ويزيد الحث الذاتي كلما زاد عدد اللفات او الفيض وحدتها ال H .

Inductance For a coaxial cable

$$a < \rho < b$$

$$H = \frac{I}{2\pi\rho} \Rightarrow B = \mu_o H \Rightarrow \quad \phi = \int_s B.ds$$

$$\phi = \int_0^d \int_a^b \quad \frac{\mu_0 I}{2\pi\rho} \;\; d\rho\; dz\;\; a_\phi = \frac{\mu_0 I d}{2\pi} \ln\frac{b}{a}$$

$$L = \frac{N\phi}{I} \quad = (per\ meter\ lenght\ ) = \frac{\mu_0}{2\pi} \ln\frac{b}{a}$$

Mutual Inductance

The mutual flux linking two coils

At 90^0 between coils it is M =0

At 0^0 between coils it is M max

$$M_{12} = M_{21} = \frac{N_1 \phi_{12}}{I_2} = \frac{N_2 \phi_{21}}{I_1} \quad (H)$$

Time Varying fields

As we know magnetic field varying with time can produce current

$$emf = -N \frac{d\phi}{dt}$$

(the -ve means that the flux act to produce an opposing flux (lenz' s law))

We can have $\frac{d\phi}{dt}$ by

1- A time-varying flux with a stationary current path
2- A relative motion between the a steady flux and closed path
3- Combination of the two

Kirchoff's Law

زمان أيام السيركتس والبراءة كنا بنقول ان كيرشوف بيقول ان محصله الجهود على مسار مغلق بتساوي صفر

$$\oint E.dL = 0$$

لكن في حاله تولد emf مستحثه معدش =0 لان هيظهر عندي نوعين من الجهد

1- ناتج عن تغير الفيض مع الزمن $-\frac{d\phi}{dt} = -\frac{d}{dt}\int_s B.ds$

2- ناتج عن الحركه $\frac{F}{Q} = E_m = v \times B$

لذا

$$emf = \oint E.dL = \int_s -\frac{\partial B}{\partial t}\, ds = \oint (\bar{v} \times \bar{B})\, d\bar{L}$$

$$transform\ voltage\ \ only\ emf = \oint E.dL = \int_S (\bar{\nabla} \times \bar{E})\, ds = \int_s -\frac{\partial B}{\partial t}\, ds$$

$$\therefore\ \ \bar{\nabla} \times \bar{E} = -\frac{\partial B}{\partial t}$$

$$Motional\ voltage\ \ only\ emf = \oint E.dL = \oint (\bar{v} \times \bar{B})\, d\bar{L}$$

Displacement Current

زمان أيام السيركتس والبراءة كنا بنقول ان المكثف مبيمرش فيه تيار وكنا بنقول ان قانون امبير بسيط وبيقولك

$$\oint H.dl = I \quad or \;\; \nabla \times H = J$$

لكن للتجارب العمليه في ال timed varying fields كان في جزء ناقص من المعادله دي وهو ال Displacement Current I_d

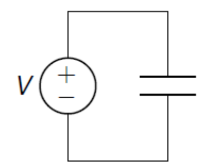
$$\oint H.dl = I + I_d = I + \int_s \frac{dD}{dt}\, ds \quad or \;\; \nabla \times H = J + J_d = J + \frac{dD}{dt}$$

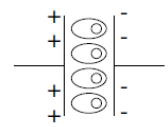
وممكن نستنتج المقدار ده $\frac{dD}{dt}$ كالاتي

$$C = \frac{\epsilon A}{d} \quad ; V = \int E.dL = Ed$$

$$\therefore I = JA = C\frac{dv}{dt} = \frac{\epsilon A}{d} * d * \frac{dE}{dt} = \epsilon \frac{dE}{dt} * A = \frac{dD}{dt} * A$$

$$\therefore \;\; J_d = \frac{dD}{dt} = \epsilon \frac{dE}{dt}$$





MAXWELL EQUATIONS

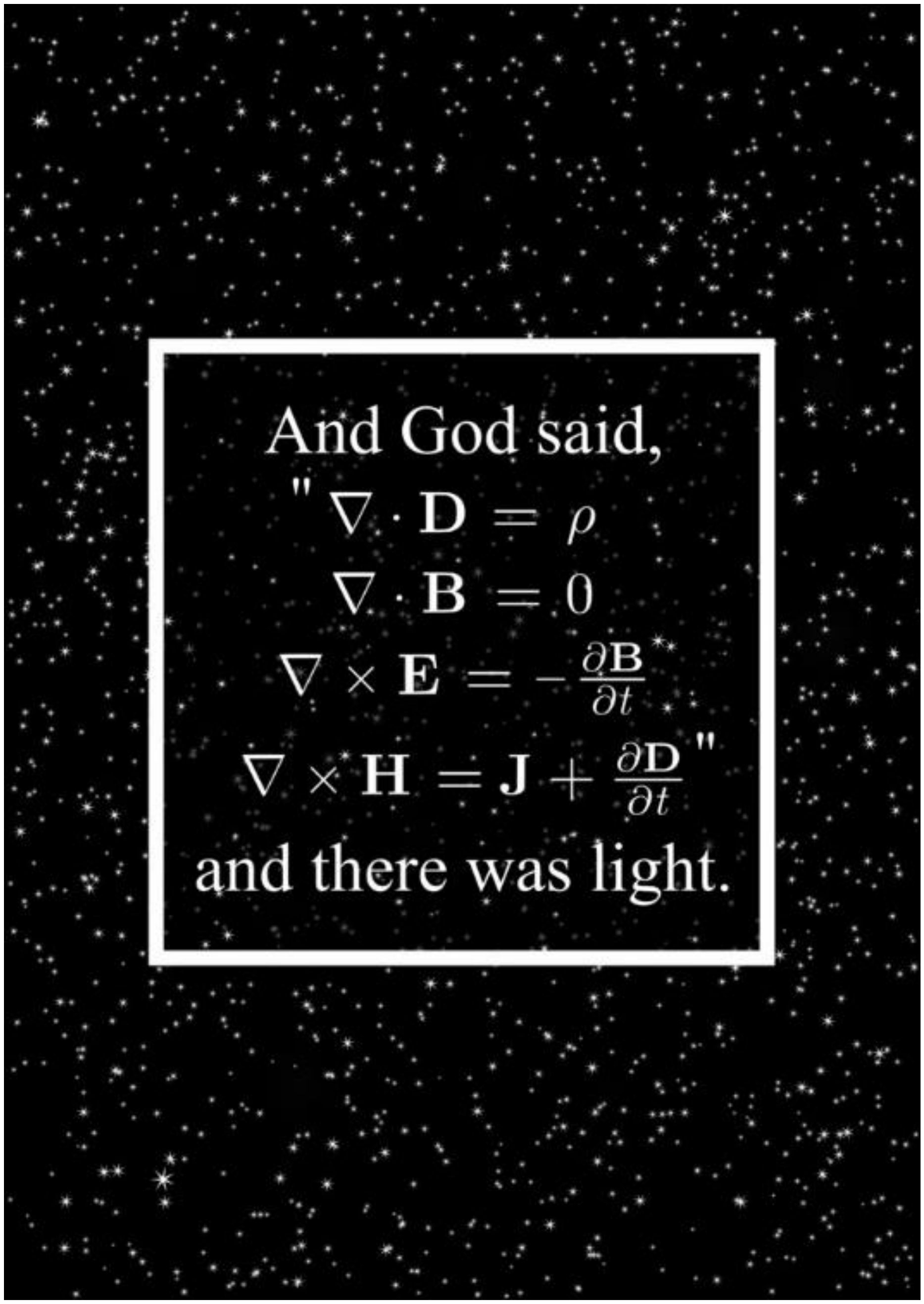
Differential Form	Integral Form
$\overline{\nabla} \times \overline{E} = -\frac{\partial B}{\partial t}$	$\oint E.dL = -\int_s \frac{\partial B}{\partial t}\, dS$
$\overline{\nabla} \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t}$	$\oint H.dL = I + \int_S \frac{\partial \overline{D}}{\partial t}\, dS$
$\overline{\nabla}.\overline{D} = \rho$	$\oint_S D.ds = \int_v \rho_v \, dv$
$\overline{\nabla}.\overline{B} = 0$	$\oint_S B.ds = 0$

Auxiliary Equations :

$$\overline{\mathbf{D}} = \epsilon \overline{\mathbf{E}} \qquad \overline{\mathbf{D}} = \epsilon_0 \overline{\mathbf{E}} + \overline{\mathbf{P}} \qquad \overline{\mathbf{P}} = \chi_e \epsilon_0 \overline{\mathbf{E}}$$

$$\overline{\mathbf{B}} = \mu \overline{\mathbf{H}} \qquad \overline{\mathbf{B}} = \mu_0 (\overline{\mathbf{H}} + \overline{\mathbf{M}}) \qquad \overline{\mathbf{M}} = \chi_m \overline{\mathbf{H}}$$

$$\overline{\mathbf{J}} = \sigma \overline{\mathbf{E}} \qquad \overline{\mathbf{J}} = \rho_v \overline{\mathbf{v}}$$


And God said,
"$\nabla \cdot \mathbf{D} = \rho$
$\nabla \cdot \mathbf{B} = 0$
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$"
and there was light.

1.27. The surfaces $r = 2$ and 4, $\theta = 30^\circ$ and 50°, and $\phi = 20^\circ$ and 60° identify a closed surface.

a) Find the enclosed volume: This will be

$$\text{Vol} = \int_{20^\circ}^{60^\circ} \int_{30^\circ}^{50^\circ} \int_{2}^{4} r^2 \sin\theta \, dr \, d\theta \, d\phi = \underline{2.91}$$

where degrees have been converted to radians.

b) Find the total area of the enclosing surface:

$$\text{Area} = \int_{20^\circ}^{60^\circ} \int_{30^\circ}^{50^\circ} (4^2 + 2^2) \sin\theta \, d\theta \, d\phi + \int_{2}^{4} \int_{20^\circ}^{60^\circ} r(\sin 30^\circ + \sin 50^\circ) dr \, d\phi$$
$$+ 2\int_{30^\circ}^{50^\circ} \int_{2}^{4} r \, dr \, d\theta = \underline{12.61}$$

c) Find the total length of the twelve edges of the surface:

$$\text{Length} = 4\int_{2}^{4} dr \; + \; 2\int_{30^\circ}^{50^\circ} (4+2) d\theta + \int_{20^\circ}^{60^\circ} (4\sin 50^\circ + 4\sin 30^\circ + 2\sin 50^\circ + 2\sin 30^\circ) d\phi$$
$$= \underline{17.49}$$

1.27. (continued)

d) Find the length of the longest straight line that lies entirely within the surface: This will be from $A(r = 2, \theta = 50^\circ, \phi = 20^\circ)$ to $B(r = 4, \theta = 30^\circ, \phi = 60^\circ)$ or

$$A(x = 2\sin 50^\circ \cos 20^\circ, \, y = 2\sin 50^\circ \sin 20^\circ, \, z = 2\cos 50^\circ)$$

to

$$B(x = 4\sin 30^\circ \cos 60^\circ, \, y = 4\sin 30^\circ \sin 60^\circ, \, z = 4\cos 30^\circ)$$

or finally $A(1.44, 0.52, 1.29)$ to $B(1.00, 1.73, 3.46)$. Thus $\mathbf{B} - \mathbf{A} = (-0.44, 1.21, 2.18)$ and

$$\text{Length} = |\mathbf{B} - \mathbf{A}| = \underline{2.53}$$

سؤال : يعطي D ويطلب Ψ .

هناك طريقتين للحل :

① $\Psi = Q_{enc} = \oint\!\!\oint \bar{D} \cdot \bar{ds}$

② $\Psi = Q_{enc} = \int \rho_v \, dV \; , \; \rho_v = \nabla \cdot \bar{D}$

** لا يخلو امتحان من هذا السؤال

Example : $\bar{D} = (4x \, , 3y^2 \, , 2z^3)$

$1 \leq x \leq 2$

$2 \leq y \leq 3$

$3 \leq z \leq 4$

Find Ψ .

$\Delta y \, \Delta z$. | sol : $\Psi = 93 \, C$

Example : $\bar{E} = -8xy \, \bar{a}_x - 4x^2 \, \bar{a}_y + \bar{a}_z$

$Q = 6C$

$B(1,8,5) \longrightarrow A(2,18,6)$

Find W

$W = -Q \int_B^A \bar{E} \cdot \bar{dL}$

طريقه الحل : تانى بعمل الحركه على ٣ مراحل
كل مرحله بنقل احداثى واحد و ثبتنا التانيين
على وضعهم السابق للحركه تانى

$\rightarrow (2, 8, 5)$

$\rightarrow (2, 18, 5)$

$\rightarrow (2, 18, 6)$

$$W = -Q\left[\int_{1}^{2} E_x \, dx \Big|_{\substack{y=8\\ z=5}} + \int_{8}^{18} E_y \, dy \Big|_{\substack{x=2\\ z=5}} + \int_{5}^{6} E_z \, dz \Big|_{\substack{x=2\\ y=18}}\right]$$

B